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Time and Dirac Observables in Friedmann Cosmologies

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Abstract A cosmological time variable is emerged from the Hamiltonian formulation of Friedmann model to measure the evolution of dynamical observables in the theory. A set of observables has been identified for the theory on the null hypersurfaces that its evolution is with respect to the volume clock introduced by the cosmological time variable.

Keywords Cosmology · Relativity · FRW · Friedmann · Time · Dirac · Hamiltonian · Observable

1 Introduction

Research in quantum gravity may be regarded as an attempt to construct a theoretical scheme in which ideas from General Relativity and quantum theory are reconciled. However, after many decades of intense work we are still far from having a complete quantum theory of gravity. Any theoretical scheme of gravity must address a variety of conceptual issues including the problem of time and identification of dynamical observables. There are many program that attempt to address the above mentioned problems including canonical quantum gravity.

It is well know that some of the issues such as time and observables in quantum gravity have their roots in classical general relativity; in such cases it seems more reasonable to identify and perhaps address the problem first in this context.The classical theory of gravity is invariant under the group of Diff ($\mathcal{M}$) of diffeomorphisms of the space-time manifold $\mathcal{M}$. To be more specific, the theory is invariant under time reparametrization and spacial diffeomorphism. This goes against the simple Newtonian picture of the a fixed and absolute time parameter. The classical theory, while itself free from problems relating to the definition and interpretation of time, contains indications of problems in the quantum theory, where the absence of a time parameter is hard to reconcile with our everyday experience. In fact, one can see that in the Hamiltonian formulation of classical general relativity, time is

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suppressed from the theory. There are many proposals for dealing with this question which generally involve a re-interpretation of the usual notion of time (see [1] for an overview of these proposals).

Identification of dynamical observable for the theory is another fundamental issue that has its roots in classical formulation of general relativity and directly related to the issue of time. The problem of evolving of a dynamical system from initial data is known as the Cauchy problem or initial value problem [2] and in General Relativity is naturally addressed using the $3+1$ ADM representation. In the Arnowitt-Deser–Misner (ADM) approach, the spatial hypersurface $\Sigma(t)$ is assumed to be equipped with a space-like 3-metric γ_{ij} (i, j runs from 1 to 3) induced from space-time metric $g_{\mu\nu}(\mu,\ \nu$ runs from 0 to 3). Einstein's equations are of course covariant and do not single out a preferred time with which to parametrise the evolution. Nevertheless, we can specify initial data on a chosen spatial hypersurface Σ, and if Σ is Cauchy, we can evolve uniquely from it to a hypersurface in the future or past. The issue of specification of initial or final data on Cauchy hypersurfaces has been discussed in many papers; for example, see [3].

An alternative approach to Cauchy problem is known as characteristic initial value problem in which one may fix the initial data on null hypersurfaces rather than spatial hypersurfaces. There are reasons to motivate us using null boundaries in formulating general relativity. For a summary one may look at the [4–10].

In addition, the approach of setting the final data on a null hypersurface is essential if we are interested in a theory such as quantum theory that observations made by a single localized observer who can collect observational data only from that subset of space-time which lies in the causal past [6].

Studying cosmological models instead of General relativity helps us to overcome the problems related to the infinite number of degrees of freedom in the theory and pay more attention to the issues arising from the time reparametrization invariance of the theory; such as the identification of a dynamical time and also construction of observables for the theory [11].

There are many general homogeneous (but anisotropic) cosmological models such as the Kantowski-Sachs models and the Bianchi models. However, in this paper we consider Friedmann-Robertson-Walker (FRW) cosmologies for simplicity. The standard FRW universe are of course one special example. In that case we assume that our universe filled with scalar massless matter field which simply has two minisuperspace coordinates, $\{a, \phi\}$, the cosmic scale factor and the scalar field. The conventional Hamiltonian formulation of this model based on Dirac and ADM procedure of general relativity is developed.

The main feature of the Hamiltonian theory of gravity is the presence of nonphysical variables and constraints due to the diffeomorphism invariance of the theory. As mentioned, This is in turn an obstacle to the problem of identification of a time parameter to measure physical quantities such as cosmological observables (the Hubble law and red shift) and the Dirac observables in the Hamiltonian description of the classical and quantum cosmologies. One of the possible solutions of these problems in the Hamiltonian approach discussed in this paper is to reduce the original theory reparametrization-invariantly by the explicit resolving of the first class constraints to get an equivalent unconstrained system. In this approach one of the variables of the extended phase space converts into the dynamic evolution parameter that plays the role of a cosmological time [6] in the theory. Thus, instead of the extended phase space and the initial action invariant under reparametrizations of the coordinate time, we obtain the reduced phase space which contains only the matter field described by the reduced Hamiltonian.

In this paper in section two, the Hamiltonian formulation of a simple reparametrization invariant model has been presented. A reduced Hamiltonian and a time variable has been emerged from this model. In section three, we apply the Hamiltonian reduction developed in section two to FRW model when a massless scalar field minimally coupled to gravity.

In section four, a discussion of Dirac observables in general relativity is given. The 'Rovelli's constants of motion' [15] have also been discussed. In section five Dirac observables on the null hypersurface of a single localized observer for FRW cosmologies is identified. These observables are similar to Rovelli's constants of motion on the null hypersurfaces [6]. The evolution of these observables is with respect to time variable obtained from massless scalar field coupled to the gravity.

2 A Simple Parametrized Model

To construct a reduced phase space with a reduced Hamiltonian for a time reparametrization invariant system let us begin with a simple toy model in classical mechanics. The case of a one dimensional motion of a particle with the action given by

$$S[x,\sigma]=\frac{1}{2}\int_{t_0}^{t_f}(N^{-2}\dot{x}^2-v(x)-N^{-2}\dot{\sigma}^2)Ndt, \tag{1}$$

where N^2 is contravariant metric and $x(t)$, $\sigma(t)$ and $N(t)$ are the independent configuration variables for the particle. With the $d\tau=Ndt$ to be the proper time interval and $v(x)$ the potential, one can rewrite the action as

$$S[x,\sigma]=\frac{1}{2}\int_{\tau(t_0)}^{\tau(t_f)}\left[\left(\frac{dx}{d\tau}\right)^2-v(x)-\left(\frac{d\sigma}{d\tau}\right)^2\right]d\tau. \tag{2}$$

With the gauge fixing, $N=1$, the dynamics is unique which is out of our interest. Without gauge fixing, i.e. allowing $N(t)$ to vary, according to the Dirac prescription the generalized Hamiltonian dynamics for the action (1) takes place on the phase space spanned by the three canonical pairs (x,p_x), (σ,p_σ) and (N,p_N).

Since σ is a dynamical variable (while τ not) and has a simple dynamics i.e. $\sigma=\alpha\tau+\beta$ (on shell), one can use σ (rather than τ) to parametrize x and also it may be considered as a clock time to make measurement.

The Euler-Lagrange equation for the dynamical variable x with respect to τ and σ are:

$$\frac{d^2x}{d\tau^2}=-\frac{\partial v}{\partial x},\qquad \frac{d^2x}{d\sigma^2}=-\frac{1}{\alpha^2}\frac{\partial v}{\partial x},$$

where α is a measurable constant. Thus, one may consider evolution of $x(\sigma)$ with respect to (measurable) clock time σ instead of τ.

The momenta associated with the dynamical variables are

$$p_x=N^{-1}\dot{x},\qquad p_\sigma=-N^{-1}\dot{\sigma},\qquad p_N=0.$$

Since the action does not explicitly depends on the variable $\dot{N}$, the vanishing momenta p_N is primary constraint,

$$p_N\approx 0.$$

The canonical Hamiltonian then is

$$H_0=p_x\dot{x}+p_\sigma\dot{\sigma}-L=\frac{1}{2}N[p_x^2-p_\sigma^2+v(x)] \tag{3}$$

with the total Hamiltonian

$$H_T = \lambda_N p_N - H_0 = \lambda_N p_N + \frac{1}{2}N[p_x^2 - p_\sigma^2 + v(x)].$$

Following Dirac procedure, to ensure that the primary constraint is preserved with time evolution, we also require that $\dot{p}_N \approx 0$. which gives us the secondary constraint

$$0 \approx \frac{1}{2}[p_x^2 - p_\sigma^2 + v(x)] = \mathcal{H},$$

and therefore the total Hamiltonian now reads

$$H_T = \lambda_N p_N + N\mathcal{H}, \tag{4}$$

where the variables λ_N and N in Hamiltonian are Lagrange multipliers. Now, the equations of motion for our system are

$$\dot{x} = Np_x, \qquad \dot{p}_x = \frac{1}{2}Nv', \tag{5}$$

$$\dot{\sigma} = -Np_\sigma, \qquad \dot{p}_\sigma = 0, \tag{6}$$

$$\dot{N} = \lambda_N, \qquad \dot{p}_N = \lambda_N, \tag{7}$$

which accompanied with the two first class constraints (FCC):

$$\mathcal{H} \approx 0, \qquad p_N \approx 0. \tag{8}$$

It is easy to check that according to Dirac procedure $x(t)$ which is the value of x for a given value of t is not eligible to be an observable since $\{H, x(t)\} \neq 0$. it means that specifying t does not identify a special point on the trajectory as the parametrization of the trajectory is not fixed. However, $x(\sigma)$ which is the value of x for a given value of σ is an eligible observable since $\{H, x(\sigma)\} = 0$. It gives us particle location when clock says for example $3:20$. Once measured and recorded, stays fixed for all time. (historical record!)

Among the dynamical variables, only p_σ is a first class variables since its Poisson brackets with the constraints vanishes. Thus, one can introduce a new canonical variables for (σ, p_σ) as

$$T_\sigma = \frac{\sigma}{p_\sigma}, \qquad p_T = \frac{p_\sigma^2}{2}, \tag{9}$$

in order to obtain a reduced Hamiltonian describing the evolution of the particle with respect to the new dynamical time variable T_σ.

In terms of the new variables the total Hamiltonian is

$$H_T = \lambda_N p_N + Np_T, \tag{10}$$

and one can divide the equations of motion into two parts: (1) For the canonical pairs (N, p_N), (T_σ, p_T) with a dependency on the Lagrange multiplier $\lambda(\tau)$,

$$\dot{T}_\sigma = N, \qquad \dot{p}_T = 0, \tag{11}$$

$$\dot{N} = \lambda_N, \qquad \dot{p}_N = -p_T, \tag{12}$$

which constrained by $p_T = 0$. (2) For the canonical pair (x, p_x)

$$\dot{x} = 0, \qquad \dot{p}_x = 0, \tag{13}$$

which have a unique solution with no constraint. The reduced Hamiltonian that governs the particle evolution in time T_σ then is

$$H(x) = \frac{1}{2}[p_x^2 + v(x)].$$

Note that although the dynamical time T_σ does not commute with the constraints and so is not a first class variable but its momenta p_T is a first class variable and so eligible to be considered as a time variable to measure the passage of time.

Alternatively one can reduce the theory in terms of the coordinate x by performing the canonical transformation on x

$$T_x = \int dx (2\Pi_x + v)^{-1/2}, \tag{14}$$

$$\Pi_x = \frac{1}{2}[p_x^2 - v] \tag{15}$$

and thus the reduced Hamiltonian that describes the evolution of the variable σ in time T_x is

$$H(p_\sigma) = \frac{p_\sigma^2}{2}. \tag{16}$$

Once again only those new canonical variables are eligible to be considered as dynamical time that their associated momenta are first class variables.

3 FRW Model with Scalar Field Minimal Coupling to Gravity

We begin with the line element for the FRW model in spherical coordinates

$$ds^2 = -N^2(t)dt^2 + a^2(t)h_{ij}dx^i dx^j, \tag{17}$$

where $N(t)$ is the lapse function, $a(t)$ is the cosmic scale factor determines the radius of the universe, and h_{ij} is the time independent metric of the three-dimensional maximally symmetric spatial sections

$$h_{ij}dx^i dx^j = \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{18}$$

of constant curvature ${}^{(3)}R(h_{ij}) = -6k$, $k = 0, \pm 1$.

Inserting the metric (17) into the action for vacuum FRW model in the natural units gives $c = h = 1$

$$S[g_{\mu\nu}] = \int \sqrt{-g}\ {}^{(4)}R d^4x = \int dt \int_{\Sigma(t)} d^3x \sqrt{-g}\ {}^{(4)}R. \tag{19}$$

By assuming the spatial homogeneity of the FRW metric the action (19) can be written as

$$S[g_{\mu\nu}] = \int dt \int 3\left(\frac{a\dot{a}^2}{N} - kNa\right)d^3x = V_{(3)} \int 3\left(\frac{a\dot{a}^2}{N} - kNa\right)dt, \tag{20}$$

where $V_{(3)}$ is the volume of the three-dimensional space of constant curvature. The momenta conjugate with the dynamical variables are

$$\Pi_a = 6N^{-1}a\dot{a},$$
$$\Pi_N = 0,$$

which $\Pi_N \approx 0$ is a primary constraint.

The canonical Hamiltonian is

$$H_0 = \left(\frac{N\Pi_a^2}{12} + kNa\right),$$

and thus the total Hamiltonian is

$$H_T = \lambda_N \Pi_N + N\left(\frac{\Pi_a^2}{12} + ka\right). \tag{21}$$

The secondary constraint is

$$0 \approx \mathcal{H} = \frac{\Pi_a^2}{12} + ka \tag{22}$$

and therefore

$$H_T = \lambda_N \Pi_N + N\mathcal{H}. \tag{23}$$

One can check that our constraints are both FCC. The number of dynamical variables are four ($a, \Pi_a; N, \Pi_N$). Since there are two FCC constraints, it means that the vacuum FRW model has no physical degrees of freedom on the classical level and only unphysical degrees of freedom propagate. So in order to have some non-trivial observables it is necessary to introduce the source matter fields.

The Einstein-Hilbert action for the FRW model for the gravity minimally coupled to a massless scalar field is given by

$$S[g_{\mu\nu}, \phi] = \int_{\Sigma(t)} \int \sqrt{-g}\left(-\frac{1}{2}\,{}^{(4)}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)dtd^3x, \tag{24}$$

which by assuming the spatial homogeneity of the scalar field in FRW metric yields

$$S[g_{\mu\nu}, \phi] = V_{(3)} \int \left[3\left(\frac{a\dot{a}^2}{N} - kNa\right) - \frac{a^3}{2N}\dot{\phi}^2\right]dt. \tag{25}$$

Given the action-integral (24) or (25) it is easy to find the canonically conjugate momentum to the dynamical variable,

$$p_a = 6N^{-1}a\dot{a}, \tag{26}$$
$$p_\phi = -N^{-1}a^3\dot{\phi}, \tag{27}$$

and

$$p_N = 0 \tag{28}$$

which is a primary constraint.

The Hamiltonian is

$$H_0 = N\left[\frac{p_a^2}{12a} + 3ka - \frac{p_\phi^2}{2a^3}\right]$$

and thus the total Hamiltonian is

$$H_T = \lambda_N p_N + \frac{N}{a^3}\left[\frac{a^2 p_a^2}{12} + 3ka^4 - \frac{p_\phi^2}{2}\right]. \tag{29}$$

By the non-degenerate character of the metric ($a \neq 0$), the secondary constraint can be redefined (choosing $N_1 = N/a^3$)

$$0 \approx \mathcal{H}_1 = \frac{a^2 p_a^2}{12} + 3ka^4 - p_\phi^2/2 \tag{30}$$

which shows the separability of the gravitational and the matter source part in the constraint. Thus, the total Hamiltonian becomes

$$H_T = \lambda_N p_N + N_1 \mathcal{H}_1. \tag{31}$$

One can check that our constraints are both FCC. The number of dynamical variables are six,(a, p_a; ϕ, p_ϕ; N, p_N). There are two FCC constraints. Thus, there are only two physical degrees of freedom on the classical level in the FRW model and only these two physical degrees of freedom propagate. According to the procedure described in the last section since a unique solution cannot be find for the equations of motion one has to implement the Hamiltonian reduction to separate the equations of motion into the physical and the unphysical ones. For this, let us introduce the new canonical variable in order to obtain the reduced Hamiltonian describing the evolution of the cosmic scalar factor a,

$$T_\phi = \int_\Sigma \frac{\phi}{p_\phi} \sqrt{^{(3)}h}\, d^3x, \tag{32}$$

$$\Pi_\phi = \frac{p_\phi^2}{2}. \tag{33}$$

In terms of the new variables the total Hamiltonian is:

$$H_T = \lambda_N p_N + N_1 \Pi_\phi. \tag{34}$$

Similarly, in here, one can separate the equation of motion into two parts: one for the canonical pairs (N, p_N), (T_ϕ, Π_T) with a dependence on the Lagrange multiplier $\lambda(\tau)$

$$\dot{T}_\phi = N V_{(3)}, \qquad \dot{\Pi}_\phi = 0 \tag{35}$$

$$\dot{N} = \lambda_N, \qquad \dot{p}_N = -\Pi_\phi \tag{36}$$

constrained by $\Pi_\phi = 0$ and second for the dynamical variables (a, Π_a),

$$\dot{a} = 0, \qquad \dot{\Pi}_a = 0 \tag{37}$$

which have a unique solution with initial values free from any constraints. The reduced Hamiltonian that governs the scale factor evolution in time T_ϕ is

$$H(a) = \frac{a^2 p_a^2}{12} + 3ka^4.$$

The equation of motion for T_ϕ derived from (32),

$$\frac{dT_\phi}{dt} = \int_{\Sigma(t)} \sqrt{-{}^{(4)}g}\, d^3x. \tag{38}$$

implies that $T_\phi(t)$ is just the 4-volume preceding Σ_t plus some constant of integration. Integration with respect to t, this means that, the change of the time variable equals the four-volume enclosed between the initial and final hypersurfaces, which is necessarily positive. This time variable, $T_\phi(t)$ may be regarded as a cosmological time variable, as it continuously increasing along any future directed time-like curve [13]. Assuming that the scalar field is spatially homogeneous and monotonically increasing along the world line observable normal to the spatial hypersurfaces, one therefore my consider $T_\phi(t)$ as a monotonically increasing function along any classical trajectory and so can indeed be used to parametrise this trajectory [6]. This describes the evolution of geometry with respect to the dynamical time constructed from scalar field.

Alternatively, by reformulating the theory in terms of scalar field one may construct a dynamical time from geometry. To find the dynamics of the scalar field we perform the canonical transformation on the scalar factor

$$T_a = \int_{\Sigma(t)} \sqrt{{}^{(3)}h}\, d^3x \int \frac{a^2 da}{(\frac{\Pi_a}{3} - a^4 k)^{1/2}}, \tag{39}$$

$$\Pi_a = a^2\left[\frac{p_a^2}{12} + 3ka^2\right]. \tag{40}$$

One can also show that the dynamical time constructed from scalar factor a is a 4-volume preceding $\Sigma(t)$.

4 Dirac Observables in General Relativity

General Relativity, like many other field theories, is invariant with respect to a group of local symmetry transformations [14]. The local symmetry group in General Relativity is the group Diff $(\mathcal{M})$ of diffeomorphisms of the space-time manifold $\mathcal{M}$.

In General Relativity, Dirac observables [12] must be invariant under the group of local symmetry transformations. The Hamiltonian constraint and momentum constraint in General Relativity are generators of the symmetry transformations, and so a function Ψ on the phase space is a Dirac observable, *iff*

$$\{\Psi, \mathcal{H}\} = \{\Psi, \mathcal{H}_i\} = 0, \tag{41}$$

at all points $x \in \mathcal{M}$, where $\mathcal{H}$ and $\mathcal{H}_i$ are Hamiltonian and momentum constraints in general relativity.

Such observables are necessarily constants of motion. They are invariant under local Lorentz rotations $SO(3)$ and Diff Σ (as well as $SO(1,3)$).

The above criteria for observables in relativity appear to rule out the existence of local observables if locations are specified in terms of a particular coordinate system. Indeed, it might appear that one would be left with only observables of the form

$$\Psi = \int \psi(x)\sqrt{-g}\, d^4x, \tag{42}$$

where $\psi(x)$ is an invariant scalar as for example R, R^2, $R^{\mu\nu}R_{\mu\nu}$, etc. While such observables clearly have vanishing Poisson brackets with all the constraints, they can not be evaluated without full knowledge of the future and past of the universe. While this may be deducible in principle from physical measurements made at a specific time, it is well beyond the scope of any real experimenter.

However, in reality, observations are made locally. We therefore ought to be able to find a satisfactory way to accommodate local observables within General Relativity. In particular, we would like to be able to talk about observables measured at a particular time, so that we can discuss their evolution. Local observables in classical or quantum gravity must be invariant under coordinate transformations. The difficulty in defining local observables in classical gravity is that diffeomorphism invariance makes it difficult to identify individual points of the space-time manifold [6, 16].

It is fairly easy to construct observables which commute with the momentum constraints. Such observables can be expressed as functions of dynamical variables on the spatial hypersurfaces. However, according to the Dirac prescription, observables must also commute with Hamiltonian constraint.

In a slightly different formalism, Rovelli addressed the problem by introducing a Material Reference System (*MRS*) [15]. By *MRS*, Rovelli means an ensemble of physical bodies, dynamically coupled to General Relativity that can be used to identify the space-time points.

Rovelli's observables can be interpreted as the values of a quantity at the point where the particle is and at the moment in which the clock displays the value t. However t itself is not an observable, even though its conjugate momentum is constant along each classical trajectory.

Rovelli's observables are constant of motion since they commute with Hamiltonian and momentum constraints, while evolving with respect to the clock time t.

Rovelli's observables are functions defined on spatial hypersurfaces. He assumes the space-time has a topology $\Sigma \times R$ where Σ is a compact spatial hypersurface and R is the real time. In order to have evolution into the future or past the spatial hypersurface must be a Cauchy hypersurface. This makes sense if the underlying space-time is assumed to be globally hyperbolic.

As discussed, one may fix the initial data on null hypersurfaces rather than spatial hypersurfaces. In General Relativity it is natural to work with a foliation of space-time by space-like hypersurfaces, as this reflects the older Newtonian idea of a 3-dimensional universe developing with time. This seems close to our experiences and is easy to visualize. In particular The approach of setting the final data on a null hypersurface is essential if we are interested in a theory such as quantum theory that observations made by a single localized observer who can collect observational data only from that subset of space-time which lies in the causal past.

5 Dirac Observables in FRW Model

In ADM formalism, the space-time $\mathcal{M}$ is assumed to be foliated by a coordinate time t. Now, suppose that the metric g satisfies FRW dynamical equations which are assumed to include a contribution from massless scalar field and we choose the foliated 3-geometry, $\Sigma(t)$ to be observer's past null hypersurface and also the space-time contains a future-directed time-like geodesic Γ representing the world-line of an observer.

Also suppose that the 4-volume time variable $T_\phi(t)$ defined in (32) instead of coordinate time t has been used to label the 3-surfaces and also the future-directed time-like geodesic Γ.

It is then possible to construct a covariantly defined geometric quantity determined by field values on $\Sigma_{T_\phi}(t)$

$$\Psi(\Sigma_{T_\phi}) = \int_{\Sigma_{T_\phi}} \psi(x)\sqrt{^{(3)}h}\,d^3x, \tag{43}$$

where $\psi(x)$ is any scalar invariant on $\Sigma_{T_\phi}(t)$ expressible in terms of h_{ij}, R^i_{jkl}, and their covariant spatial derivatives.These quantities are called world line Γ-observables [13] for FRW model.

The so called Γ-observables then have vanishing Poisson brackets with any Hamiltonian H, equation (31), which generates time translations of $\Sigma_{T_\phi}(t)$ along Γ. The observables $\Psi(\Sigma_{T_\phi})$ do not have vanishing Poisson brackets with the Hamiltonian constraint $\mathcal{H}_1$, since the prespecified foliation is not invariant under local time evolution [17].

If we define new quantities, $\Psi_{T_\phi}(\Sigma_{T_\phi})$; the value $\Psi(\Sigma_{T_\phi})$ at a certain time T_ϕ, then these quantities have vanishing Poisson brackets with the Hamiltonian constraint, $\{\Psi_{T_\phi}(\Sigma_{T_\phi}), \mathcal{H}_1\} = 0$, and can be called 'evolving constants of motion'. These observables are the same as Rovelli's constants of motion in a sense that they are genuine Dirac's observables. The evolution of these observables is expressed in terms of the dynamical variable $T_\phi(t)$, whose conjugate momenta, is a first class constraint.Similarly, the dynamical time $T_\phi(t)$ in the new labeling of 3-surfaces is not a Dirac observable although its conjugate momenta is constant along the world line.

Alternatively, using (39) it is also possible to construct a covariantly defined matter quantity determined by the scalar factor values on $\Sigma_{T_a}(t)$

$$\Psi(\Sigma_{T_a}) = \int_{\Sigma_{T_a}} \psi(\phi)\sqrt{^{(3)}h}\,d^3x, \tag{44}$$

where $\psi(\phi)$ is any scalar invariant on $\Sigma_{T_a}(t)$ expressible in terms of ϕ, and its covariant spatial derivatives. These quantities are also called world line Γ-observables.

In summary we have seen that an explicit time variable has been emerged in FRW model from gravity coupled to the massless scalar field, interpreted as a cosmological time, and can be used by observers as a clock to measure the passage of time. A set of 'evolving constants of motion' has been constructed by using the dynamical time variable emerged from scalar field or scalar factor which set the condition on the Γ-observables.

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